Grade 11/12 Math Circles

April 3, 2024

Primality Testing and Integer Factorization - Problem Set

1. Calculate 137×73 mod the following m:

$$10, 100, 1000, 10000, 100000.$$

2. Calculate $2^{35}, 2^{70}, 2^{140}, 2^{280}$, and $2^{560} \pmod{561}$. Hint: $2^{32} = (2^{16})^2 = ((2^8)^2)^2$ and so on. Then you can calculate 2^{35} as $2^{32} \times 8$.

3. Are 91 and 169 coprime? Are 97 and 99 coprime?

4. Calculate inverses of 1, 2, 4, and 14 mod 15. Hint: $14 \equiv -1 \pmod{15}$.

5. Calculate $\phi(10)$, $\phi(15)$, and $\phi(17)$.

6. Calculate $\phi(210)$, $\phi(216)$, and $\phi(257)$. Hint: 257 is prime.